1982

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Wood, Gregory James.

Massachusetts Institute of Technology

ANALYTICAL PREDICTION OF FAILURE LOAD FOR A CIRCULAR BULKHEAD WITH HORIZONTAL AND VERTICAL STIFFENERS

by

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(1968)

SUBMITTED TO THE DEPARTMENT OF OCEAN
ENGINEERING IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREES OF

OCEAN ENGINEER

and

MASTER OF SCIENCE IN

NAVAL ARCHITECTURE AND MARINE ENGINEERING

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1982

ANALYTICAL PREDICTION OF FAILURE LOAD FOR A CIRCULAR BULKHEAD WITH HORIZONTAL AND VERTICAL STIFFENERS

by

Gregory James Wood

Submitted to the Department of Ocean Engineering on 7 May 1982 in partial fulfillment of the requirements for the Degrees of Ocean Engineer and Master of Science in Naval Architecture and Marine Engineering.

ABSTRACT

A Computer program to determine the ultimate failure load of a circular bulkhead with horizontal and vertical stiffeners is presented. The circular plate is treated as a grillage with the plating acting as an effective flange on the stiffener. The grillage is composed of a combination of transverse and longitudinal members, however, the spacing between nodal points is not uniform. The plate-stiffener combination is subjected to a uniform hydrostatic pressure that is modelled as equal point-forces at each of the interior nodes. A matrix system of inequality constraints (on maximum moment values) and nodal point equilibrium equations is established and the applied point-force is maximized using a Linear Programming maximization routine (ZX3LP) from the International Mathematical and Statistical Library (IMSL). A detailed explanation of the computer program is presented.

The ultimate failure load for the unsupported segments of plating (between stiffeners) is investigated. A brief review of the use of the Upper and Lower Bound Theorems is presented.

Results from the computer program are compared to experimental data for similarly stiffened circular plating subjected to a uniform pressure until collapse.

Thesis Supervisor: Dr. Joao Manuel Gomes de Oliveira

Title: Associate Professor of Ocean Engineering

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CHAPTER 1 INTRODUCTION

BACKGROUND

In the last decade, the demand for new and more efficient modes of marine transportation has forced naval architects and marine engineers to search for practical and reliable methods of structural analysis which would take full advantage of the inherent strength of the materials of construction in providing a safe design with a minimal structural weight.

Elastic design methods provide an accurate and reliable technique for predicting structural deformations resulting from applied design loadings whenever the material behavior is linear elastic. However, in applications where the total deformation is not a limiting factor, elastic analysis methods do not provide an effective technique for prediction of the ultimate failure load of a given structural design.

In an attempt to take full advantage of the inherent ductility of structural steels, a design methodology was developed which treats load levels exceeding that necessary to reach initial yield stress (elastic limit). This methodology, called plastic limit analysis, is not useful for predicting local stress levels or deformations, but is a reliable and effective method of determining the ultimate collapse load. In those cases where structural deformation is not a limiting factor, or where the ultimate load-bearing capacity of a given structure is a primary concern, plastic limit methods of analysis provide the structural engineer with an effective tool for accurately determining the load limit of a given structure. If the designer requires additional information, such as local stresses and deformations, or if material strain hardening or finite deflections are an important consideration, then a numerical analysis using an incremental theory of elasticity will need to be performed {1}.

A detailed review of available literature concerning plastic limit analysis of structures with stiffened and unstiffened plating indicated a great

deal of recent interest in applying "plastic" techniques to structural analysis and design, with a view toward structural optimization. By far, the greatest majority of stiffened-plating applications developed utilizing plastic limit analysis dealt with a class of problems that could most generally be categorized as stiffened rectangular plating with uniformly spaced stiffeners.

An unstiffened circular plate has been analyzed using plastic limit analysis {2} with a linear programming technique and the "Lower Bound" Theorem (to be discussed in detail). Similarly, a stiffened circular plate was studied using elastic analysis to optimize the plate-stiffener combination {3}. In this analysis, however, the stiffeners were a radial and circumferential system of stiffeners (axisymmetric) which were compatible with the axisymmetric circular plate, and the stiffened circular plate was optimized by maximizing the load per unit deflection.

In the field of submersible vehicle design, a cylindrical hull form provides the best compromise between competing criteria of adequate hull strength,

maximum internal volume, and minimum hydrodynamic drag. As a result, submersible vehicle bulkheads are essentially stiffened circular plates. Due to the requirement for minimizing fabrication cost, the bulkhead stiffening system is composed of a grid network of horizontal and vertical stiffeners, not radial and circumferential stiffeners. Thus, the as-constructed submersible vehicle bulkhead cannot be analyzed by an adaptation of available elastic (or plastic) axisymmetric, orthotropic plate analysis methods. Also, ultimate failure is the design criteria, without any limit on deformation.

In their paper]{4}, Palermo and Bart applied the concept of plastic analysis to the as-constructed submersible vehicle bulkhead. The method detailed in this paper combined the concept of a plastic hinge and elastic grillage analysis to determine the required scantlings for the main horizontal girder. However, the vertical stiffeners were analyzed individually with the main horizontal girder assumed to be a fixed support for each half of the vertical stiffener. More recent methods allow for plastic analysis of the grillage as a whole.

This thesis will attempt to analyze the stiffened circular bulkhead as a grillage, adapting plastic grillage limit analysis techniques developed by Hodge {5} for a rectangular grillage. The net result will be an interactive computer program (CIRCPLAT) designed to be inexpensive to run and simple to use, providing a reasonably accurate, yet conservative estimate of the collapse load of a user-defined, stiffened, circular plate.

The results of the CIRCPLAT analysis will be compared to actual bulkhead failure test data.

CHAPTER 2 PROBLEM DEFINITION

FORMULATION OF THE PROBLEM

The basic problem under consideration is to determine the failure load of a built-in, circular, stiffened bulkhead using horizontal and vertical stiffeners when it is subjected to a uniform, lateral, hydrostatic pressure. To find an accurate solution in the simplest, most efficient manner, it is first necessary to decide what is the most effective way to model the problem. Reference {6} provided a recent, well researched compendium of current methods of gross panel analyses along with the inherent assumptions and limitations of each method, and a comparison of their relative accuracy. Figure 1 provides a brief summary of the results of comparison of analyses methods given by reference {6}. Reference {6} also indicated excellent results using plastic limit analysis. Based on a very favorable comparison between the finite element program results and the "grillage analysis",

and, the close correlation between both methods and experimental data, it appeared to be highly advantageous to model the stiffened circular plate as a grillage of intersecting beams with an effective width of plating acting as a flange to the stiffener. With this model, it should be possible to develop a computer program that is relatively inexpensive to run, yet sacrifices little or nothing in accuracy for determination of the failure load.

		Total Stress (psi) Centerline Longitudinal Stiffener at Fixed Boundary End	Percentage Difference **
Orthotropic Plate Analysis	1st Order	23,320	-11%
	2nd Order	23,585	-10%
Finite Element SOLID SAP	Beam & Plate Elements	26,285	+.04%
	Orthotropic Plate Elements	22,933	-12%
Finite Element ICES STRUDL		26,186	----
Grillage Analysis		25,721	-2%
Simple Beam Theory		30,184	+15%

** Percentage difference with respect to ICES STRUDL Results

Figure 1. Stiffened Panel Analysis Method Comparison.

GRILLAGE MODEL: ASSUMPTIONS

The use of the grillage model in conjunction with

plastic limit analysis involves several approximating assumptions:

1. The in-plane compressive loading is small compared to the lateral loading and its effect is negligible.

2. The influence of the shear force on the formation of the plastic hinge is negligible.

3. The torsional rigidity of the stiffeners is small and its effect is negligible.

4. The effect of the plating can be accounted for by incorporating an "effective breadth" of plating as a flange on the stiffener.

5. Plane sections remain plane in the plastic range.

6. Deflections are infintessimal.

7. The material is elastic-perfectly plastic;

8. Drucker's Postulate is valid.

Reference]{5} discusses the validity of assuming that the effect of in-plane stresses are negligible. Experimental evidence indicates that if the in-plane stress is less than 15 percent of the compressive yield stress, the effect of the axial force can be neglected. Current submersible pressure hull design, utilizing a ring-stiffened cylinder, provides adequate support against hull compressive effects sufficient to ensure

that compressive axial stresses in the bulkhead stiffeners will not exceed 15 percent of yield stress. References {4} and {6} contain correction factors that can be applied to the stiffeners' plastic section moduli to account for the effect of in-plane stresses above 15 percent of yield. These correction factors were not incorporated in the CIRCPLAT program. The program could easily be modified to include these corrections if large axial forces were to be included in the analysis.

The assumption that shear force is negligible is a reasonable one for long, slender beams. For short, deep beams, a correction factor must be applied to the section modulus. To study the relative magnitude of shear and bending stress in a beam, consider a rectangular, prismatic beam with a single concentrated load as shown in Figure 2: (Reference {6})

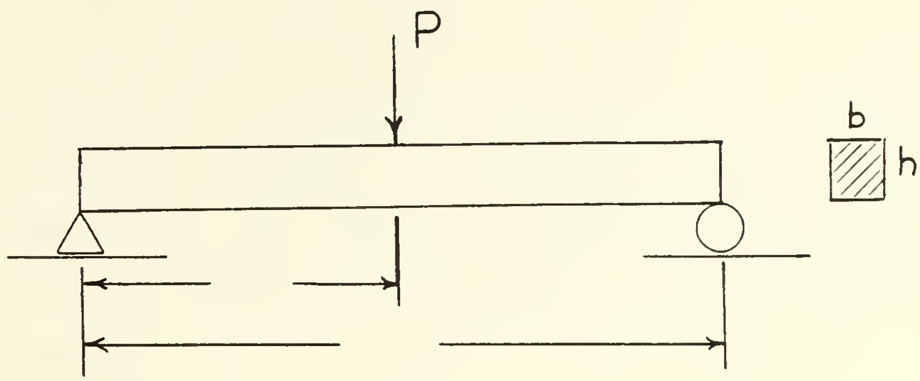


Figure 2. Rectangular Beam with Concentrated Load.

The maximum bending moment, which occurs at midspan is: $M=PL/4$. The maximum bending stress will be at the same point:

$$\sigma max = Mc/I = (3/2)(PL/bh^2).$$

The shear force is constant at $P/2$ between each support and the load P. The formula for maximum shear stress in a rectangular beam is:

$$\tau max=(3/4)(P/bh).$$

Then $\sigma max/\tau max = 2L/h$. Thus, we can see that when L is many times (10 or more) greater than h, the magnitude

of the maximum shear stress as compared to the maximum bending stress is negligible.

Reference]{7} discusses the practical significance of torsional rigidity in grillage analysis, and concludes that the effect of torsion on the plastic moment for I-beams is 0.1 percent, and is therefore negligible.

The question of incorporating an "effective" breadth of plating as a flange on the stiffener is one that appears to have a number of answers as to how much breadth to include as "effective." Reference {4} used a value equal to the stiffener spacing, up to a maximum of 60t, where t=plate thickness. Reference {7} utilized a formula developed by Schade:

$$B = 1.1/(1+2/r^2)$$

where B is the fraction of stiffener spacing that is effective, and r is the ratio of the distance between points of zero bending moment and the plate's breadth. In reference {8}, extensive experimental research by Kendrick was performed specifically to determine the effective breadth of plating to be included as a flange to the stiffener when calculating the plastic section

modulus. The results of this research showed that experimental plastic moments agreed quite closely with the theoretic values calculated by assuming all the plating effective as a flange to the stiffener. For this reason, the plastic section modulus calculation incoporated in the CIRCPLAT program utilizes the full stiffener spacing as the "effective" breadth, with no maximum limit (such as 60t).

The remaining assumptions are the standard assumptions utilized in deriving the equations for plastic limit analysis.

PLATE MODEL ASSUMPTIONS

As an additional consideration, it is necessary to consider the possibility of plate failure between stiffeners. Because this is a circular plate intersected by horizontal and vertical stiffeners, the resulting sections of unsupported plate created come in a variety of shapes (see figure 3). Plastic limit

analysis can similarly be applied to these plate sections to determine their ultimate failure load.

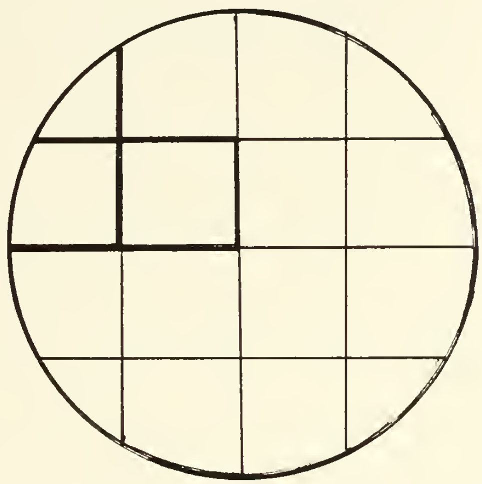


Figure 3. Stiffened Circular Plate Geometry.

In applying plastic limit analysis to thin plates it is assumed:

1. Elastic-perfectly plastic material
2. No in-plane loads
3. Infinitessimal deflections

4. No shear effects

5. Johansen's yield surface applies

Reference {9} details the use of Upper and Lower Bound Theorems of Plasticity for determining plate failure load subject to the above assumptions.

CHAPTER 3 STRUCTURAL ANALYSIS

BASIC PRINCIPLES

Reference {1} presents an excellent development of the theory of plastic analysis of structures. In understanding plastic failure analysis, it is first necessary to understand the concept of the plastic hinge. Consider pure bending of a prismatic beam of rectangular cross-section subject to an applied moment. Coordinate axes and sign convention are positive as indicated in figure 4.

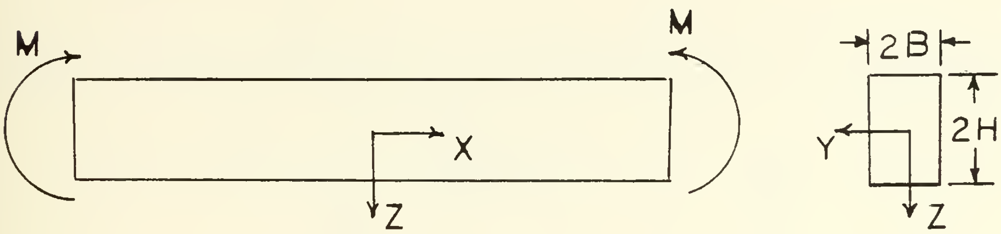


Figure 4. Rectangular Beam subject to pure bending

The bending moment is given by:

$$M=2B\int_{-H}^{H} z\,\sigma_x(z)\,dZ$$

The maximum elastic bending moment will occur when the stress at the outer fiber reaches yield. Then the moment in this case becomes:

$$Me=(4/3)(BH^2\sigma_o)$$

If the applied moment is increased still further, the outer fibers will begin to deform plastically, and the elastic to plastic interface will begin to move

toward the neutral axis until, eventually, k=0 (see figure 5.)

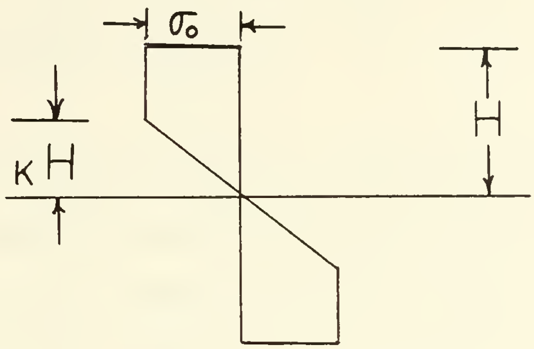


Figure 5. Elastic-Plastic Stress Distribution

At the point where k=0, the beam can not support any further increase in applied moment, and a plastic hinge is said to have developed. This fully plastic moment, denoted by Mo, for a rectangular section, is:

$$Mo = 2\sigma_o BH$$

In the partially plastic range between Me and Mo, the deformation is theoretically controlled by the assumption that plane sections remain plane.

A yield hinge, then, has the following properties:

1. If the applied moment is less than Mo, the yield hinge will transmit the full moment without allowing any rotation.

2. If the applied moment is equal to Mo in magnitude, the yield hinge will transmit the full moment, but will permit rotations of arbitrary magnitude in the direction of the applied moment.

3. The yield hinge will not transmit any moment greater than Mo in magnitude.

With the concept of a yield hinge defined, failure of a structure occurs when sufficient yield hinges are formed to allow infinitessimal motion of all or part of the structure. The existance of sufficient yield hinges is termed a "mechanism".

Several important concepts involved in the idea of a failure mechanism are exemplified in the case of a simple, indeterminate structure under a distributed load. Consider the case of a beam fixed at one end,

simply supported at the other end, and under a uniformly distributed load P, as in figure 6.

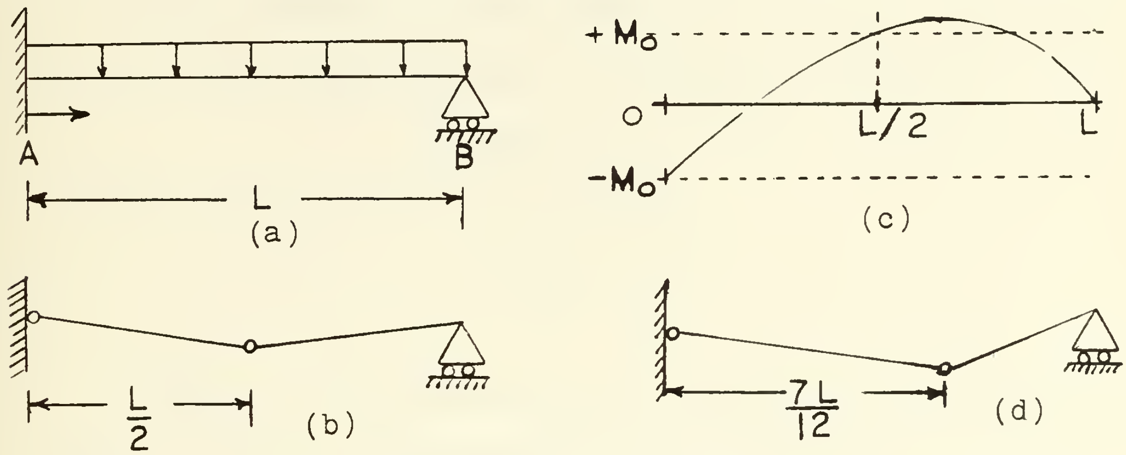


Figure 6. Beam with Distributed Load

First, it can be seen that the problem is indeterminate, because there are 3 unknown reactions and only 2 equations of equilibrium (horizontal force equilibrium is implicit). From elastic analysis, we know that the shape of the moment distribution is that of figure 6.c. Thus, we know that a plastic hinge will form at the fixed end, and one will form somewhere near the center. As soon as the first hinge is formed, the problem becomes determinate since we have one less

unknown, leaving 2 unknowns and 2 equilibrium equations.

For example, if the first hinge forms at the fixed end, then summing moments about point A gives:

$$-Mo+R_BL-PL\ /2 = 0$$

Summing y-directed forces gives:

$$R_A+R_B = PL$$

These two equations can be solved for both R_A and R_B. Although the problem is now determinate, we do not yet have a mechanism. The plastic hinge at A would allow rotation, but the simple support at B still prevents it, so a second hinge must form somewhere near the center of the beam. These two hinges now form a collapse mechanism. Plastic hinge rotation can proceed unimpeded.

UPPER BOUND THEOREM

Continuing with the example of Figure 6, assume

that the second hinge forms exactly at the center of the beam. The internal work of deformation is the product of the plastic moment times the angle through which it rotates. For the case of the beam in Figure 6, if the hinge at the fixed end rotates by Θ, the center hinge rotates by 2Θ. Work is independent of direction of rotation. Then

$$Wi=M_O\theta+M_O(2\theta)=3M_O\theta.$$

External work is the product of applied force times the distance moved. For a distributed load, the average distance moved is $1/4(L^2\theta)$. If the applied load is large enough to do sufficient external work to equal the required internal work of the collapse mechanism, then collapse will occur. Therefore, $[P/4](L^2\theta)=3M\theta$ will yield the value of P required for our assumed mechanism. This result, $P=12M_O/L^2$, is the required distributed load to cause the "assumed" collapse mechanism to occur. If the assumed collapse mechanism were the exact collapse mechanism, the value for P would be the exact value to cause collapse. Looking at Figure 6c., the moment distribution shows that the maximum moment occurs to the right of the center of the beam. As a result, the actual plastic

hinge will form to the right of the center. As a result, the collapse load, P, determined above, is not the exact collapse load. The Upper Bound Theorem essentially states that the failure load determined by an assumed collapse mechanism that is not the exact collapse mechanism will be an upper bound to the exact failure load. Thus, the load, $P^u = 12M_o/L^2$, is greater than the actual collapse load, and is an Upper Bound.

LOWER BOUND THEOREM

Remembering that once the first plastic hinge forms, the problem becomes determinate, the following equations can be written:

$$M(0) = R_B L - PL^2 = -Mo$$

$$M(Z) = R_B(L-Z) - P(L-Z)^2/2 = Mo$$

$$M'(Z) = -R_B + P(L-Z) = 0$$

where Z is the point at which the second plastic hinge forms. With 3 equations and 3 unknowns, the reaction R, distance Z and load P can be determined. The result is:

$$R = 4.8284\ M_O/L$$

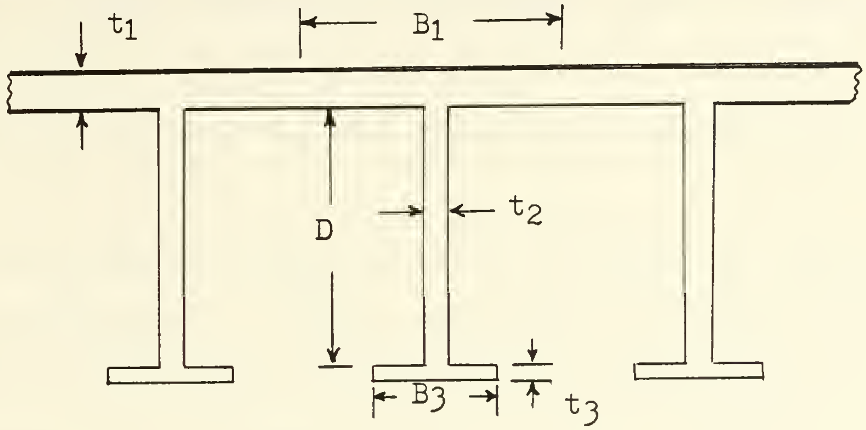
$$P = 11.65685\ M_O/L^2$$

$$Z = 0.58579\ L$$

From the moment curve of Figure 6c, it can be seen that if the moment at x = 0 is -Mo and the moment at $x=0.58579 * L$ is +Mo, then the moment can not exceed Mo in magnitude anywhere on the beam. This, then, constitutes a statically admissible moment distribution. The Lower Bound Theorem essentially states that if a statically admissible moment distribution can be found that satisfies equilibrium and the boundary conditions, and does not violate the yield criterion (in this case, $M(x) \leq Mo$ for all x), then the load P that corresponds to this moment distribution is a lower bound to the exact solution. For the example of Figure 6, the value of $P^L=11.65685$ is a lower bound, but in this case it also corresponds to the exact solution. If the upper bound load, P^U, were determined at the same point on the beam, the same P would result, $P^U=P^L$, indicating the exact solution was obtained.

EFFECTIVE PLASTIC MOMENT OF A STIFFENER

Several formulations are available to calculate the effective plastic moment of a given I-beam, yet the problem of incorporating a general formula in a computer program for an I-beam, where the user specifies the thickness and breadth of the upper flange (plate thickness and stiffener spacing) separately from the remaining stiffener scantlings, presents an interesting subtlety. The program must be able to distinguish between the case where the neutral exis lies in the web and the case where the neutral axis lies in the plate. A simple, accurate, and effective technique to accomplish this is given in reference {7}. Typical stiffened plate geometry is shown in Figure 7.



$$r_1 = Dt_2/B_1t_1$$

$$r_2 = B_3t_3/B_1t_1$$

$$r_3 = t_1/D$$

$$r_4 = t_3/D$$

Figure 7. Stiffened Plate Geometry for Plastic Section Modulus

When $r_1+r_2>1$, the plastic neutral axis lies in the web and the formula for the plastic section modulus, Z, is:

$$Z=B_1Dt_1\{r_1(r_1+2r_3+2)+2r_1r_2(r_4+1)-(r_2-1)^2\}/4r_1.$$

When $r_1+r_2<1$, the plastic neutral axis lies in the upper flange (plate) and the plastic section modulus is given by:

$$Z=B_1Dt_1\{2r_1+4r_2+r_3(1+2r_1+2r_2)-r_3(r_1+r_2)^2+2r_2r_4\}/4.$$

From chapter two, the value of effective breadth to be incorporated in the calculation of plastic section modulus was shown to be the full stiffener spacing {8}, without an arbitrary cutoff value. CIRCPLAT sets B equal to stiffener spacing for all calculations.

GRILLAGE COLLAPSE LOAD

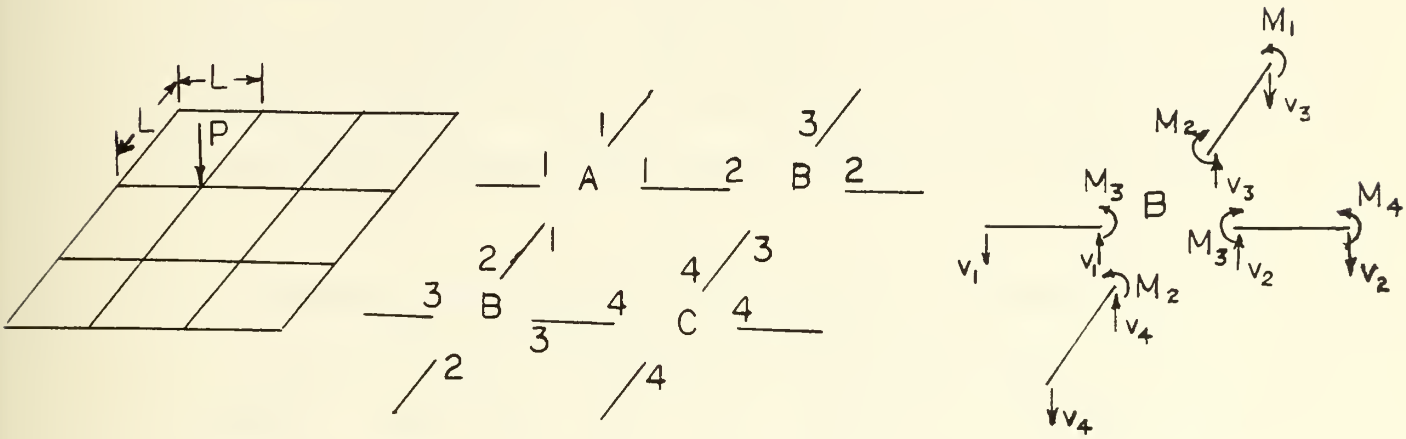
In reference {10}, Hodge applied the Upper Bound Theorem to a rectangular grid (hereafter called a "grillage") in order to determine its collapse load. In the method development, it was assumed that concentrated loads were applied at the joints (or nodes), of the grillage, although the method could be made applicable to distributed loads. Having assumed a collapse mode and determined the corresponding upper

bound collapse load, it is necessary to construct a statically admissible bending moment distribution to verify that the assumed collapse mode is feasible. If there are no other collapse modes that would yield a lower collapse load and the moment distribution is statically admissible (the absolute value of all moments are at, or lower than, the plastic yield moment), then the correct collapse load has been determined. Reference {10} contains numerous examples of the above procedure.

This method can be carried out with little difficulty for regular rectangular grillages, where the loads at all nodes are equal. A regular rectangular grillage is one where the spacing is uniform between longitudinal beams, and the spacing is also uniform between transverse beams. In cases where these requirements are not met, it becomes difficult to predict a deformation pattern that will result in an improved upper bound. Moreover, the assumed deformation pattern may not have a statically admissible moment distribution.

Reference {5}, by Hodge and Malone, presented a feasible method to overcome these difficulties in obtaining a correct collapse load. The proposed method involves taking a Lower Bound approach using Linear Programming. The load associated with a statically admissible moment distribution is, by the Lower Bound Theorem, a lower bound to the true collapse load. If the load can be maximized while the moment distribution is maintained statically admissible, the maximum lower bound load should be the correct collapse load. Linear Programming, explained in Appendix B, provides a simple, computer method to maximize the load, subject to the constraints of static admissibility and the equations of equilibrium.

The grillage of Figure 8 will be discussed in detail as an example of the use of the Lower Bound Theorem in conjunction with Linear Programming.



(a) Grillage with load (b) Symmetric node arrangement (c) Detail of Node B

Figure 8. 2x2 Grillage with Non-Uniform Loading.

For this problem the beams are simply supported at all boundaries. The plastic yield moment of the horizontal and vertical beams is Mo. Moments are positive when they produce compression in the top fibers.

From Figure 8b we can see that there is symmetry about a line through nodes A and C. This symmetry can be used to more effectively solve the problem. As a result of symmetry, the number of unknown moments is reduced from 8 to 4. The numbers on the ends of the beam spans indicate the unknown moments in Figure 8b.

Because the problem of plastic collapse is only concerned with the statically admissible moment field, support reactions are not required to be solved for and, as a result, are not listed as unknowns.

Since the loads are applied at the nodes, there are no midspan loads and so for each span, the shear forces at the ends must be equal and opposite. Moment equilibrium at node B for each span means that $V_1 = (M_3 - 0)/L$, $V_2 = (M_3 - M_4)/L$, etc, from Figure 8c. The sum of the shear forces at each node must equal the applied load at that node. For node B, the result is:

$$(2M_3 + 2M_2 - M_4 - M_1)/L = 0$$

$$\text{or } 2M_3 + 2M_2 - M_4 - M_1 = 0$$

Applying similar reasoning to the other two independent nodes:

$$4M_1-2M_2-P = 0$$

$$4M_4-2M_3 = 0$$

Notice that there are four unknown moments, and only three independent equilibrium equations.

In addition to the equations of equilibrium, the linear programming problem requires the equations of constraint. Each of the moments is constrained to be equal to or less than the plastic yield moment, however, this moment can be positive or negative. The linear programming constraint values must all be positive. In order to meet this requirement, a new variable is created, $X_i=M_i+Mo$ i=1,2,3,4 so that while $-Mo\leq M_i\leq +Mo$, $0\leq X_i\leq 2Mo$. Therefore, X_i is constrained to positive values. The collapse load is the fifth unknown, called X_5.

Substituting $M_i=X_i-Mo$ into the three equilibrium equations the result is:

$$-X_1+2X_2+2X_3-X_4 = 2Mo$$

$$4X_1-2X_2 \quad -X_5 = 2Mo$$

$$-2X_3+4X_4 = 2Mo$$

The load, which is the function to be maximized can be defined by a separate equation (called the objective function):

$$X_5 = P$$

By the above procedure, all of the requirements for establishing a linear programming problem have been met. The next step is to format the information to facilitate input to the Linear Programming routine. The required input format is a function of the linear program to be utilized. At Massachusetts Institute of Technology, one of the standard linear programs available in the International Mathematical and Statistical Library (IMSL) is called ZX3LP (see reference {14}). This program was utilized by CIRCPLAT and is explained fully in Appendix B.

ZX3LP requires the input to be arranged in 3 matrices. Matrix A contains the coefficients of the unknown variables in the equations of constraint, followed by the coefficients of the unknown variables in the equilibrium equations. Matrix B contains the values from the right-hand side of the constraint equations, followed by the values from the right hand side of the equilibrium equations. Matrix C contains the coefficients of the unknown variables in the objective

function. Matrices A,B, and C for the sample problem of Figure 8 are shown in Figure 9.

A Matrix

$$
\begin{array}{ccccc}
X_1 & X_2 & X_3 & X_4 & X_5 \\
\end{array}
$$

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
-1 & 2 & 2 & -1 & 0 \\
4 & -2 & 0 & 0 & -1 \\
0 & 0 & -2 & 4 & 0
\end{bmatrix}
$$

B Matrix

$$
\begin{bmatrix}
2Mo \\
2Mo \\
2Mo \\
2Mo \\
2Mo \\
2Mo \\
2Mo
\end{bmatrix}
$$

C Matrix $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

Figure 9. Linear Program ZX3LP Input Matrices.

The primary solution of the ZX3LP program is contained in array PSOL(N). The values in array PSOL(N) are the values of $X_1, X_2, X_3, \ldots\ldots, X_N$. For the problem of Figure 8, the values of PSOL(1) through PSOL(4) must be adjusted by the value Mo to give the actual nodal moments. This is because $M_i = X_i - Mo$. The value of the collapse load is given by PSOL(5) for this case.

REPLACEMENT THEOREM

The derivation of a method for determining the collapse load of a grillage has, up to now, been for grillages with concentrated loads at beam intersections. In extending this method to encompass the problem of a uniformly loaded circular plate, it is necessary to discuss the correlation between the concentrated load problem and the case of the uniform load. Reference {10} contains a discussion and proof of the Replacement Theorem. The essence of the Replacement Theorem is that if a given beam span of constant plastic yield moment, Mo, is acted upon by a distributed load whose direction does not change sign on that span, the

span is not made stronger if the distributed load is replaced by equipollent concentrated loads. That is, the collapse load of a span under a set of concentrated loads will be less than that for the same span under the equipollent distributed load.

An equipollent force system is defined in reference {12} as one in which:

1. The resultant of the forces of one system is equal to the resultant of forces of the other system.

2. The sum of the moments of one system about an arbitrary base point is equal to the sum of the moments of the other system about the same point.

Using the above theorem, if the uniform load on the grillage is replaced by an equipollent system of concentrated forces at nodal points, then the method of grillage analysis just developed, which computes a "best" lower bound collapse pressure for concentrated forces, can be applied directly to compute a conservative "best" lower bound for a uniformly distributed load.

Looking at the section of stiffened plate in Figure 10, it can be seen that the pressure P distributed over an area a x b can be replaced by a concentrated force of magnitude P(a x b).

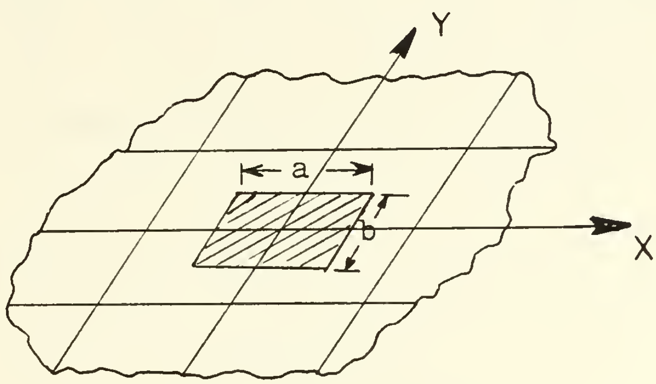


Figure 10. Section of a Stiffened Plate

If the spacing is uniform in the x-direction and is also uniform (not necessarily the same) in the y-direction, then the area a x b will be the product of the stiffener spacing in the x and y direction. From this, it can be seen that for a regular rectangular grillage, the concentrated force at every node will be the same. In the case of a grillage stiffened circular plate, where the stiffeners are equally

spaced along the main diameters (x and y directions), this same replacement process works exactly the same for nodes not near the circumference of the circle. The replacement process for nodes near the circumference is not exact, as can be seen in Figure 11. There are nodes where the replacement force will be too high, and nodes where the replacement force will be too low, but overall, the area of the circle is greater than the area associated with all of the nodes.

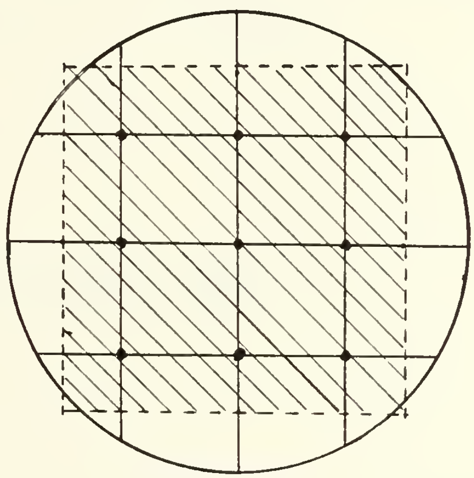


Figure 11. Top View of a Stiffened Circular Plate.

It is necessary to ensure that the estimate provided by the application of the Replacement Theorem remains conservative for the case of the stiffened circular plate. In CIRCPLAT, the forces constrained to be equal at all of the nodes. Then, after the collapse load is determined, the collapse pressure is calculated by dividing the sum of the nodal forces by the total area of the circular plate [P=(Sum of F)/A . Since the area in the denominator is greater than that associated with the nodal forces (the rectangle of Figure 11.), the pressure will be conservative. The accuracy of this method is improved as the number of beams in the grillage is increased.

UNSUPPORTED PLATE COLLAPSE LOAD.

Calculating the collapse load of the grillage does not provide any information about the possibility of collapse of the sections of plating between stiffeners. To be complete, analysis of the stiffened plate must include a determination of the collapse pressure of the unsupported plate segments. Figure 12 indicates the variety of shapes possible for

unsupported segments of a stiffened circular plate. In order to simplify the collapse load calculation, use is made of a corollary to the Lower Bound Theorem discussed in reference {13}. This corollary states that you cannot decrease the collapse load of a structure by increasing the strength of any part.

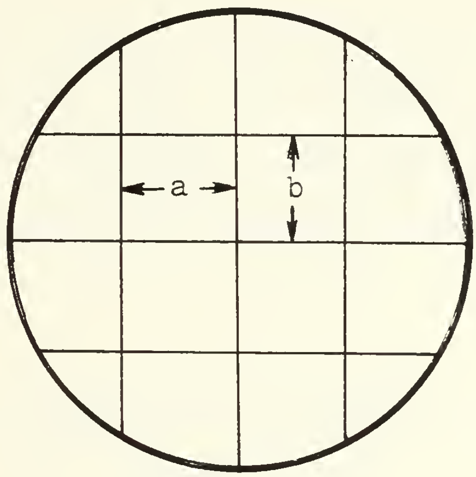


Figure 12. Unsupported Plate Segments of Stiffened Circular Plate.

In Figure 12 we can see that if the rectangular sections of unsupported plating are a x b, the unsupported sections

along the perimeter of the circle are less than a x b. These sections can be considered to be rectangles of dimensions a x b with a wall built in to the rectangle for increased support. Here, invoking the corollary to the Lower Bound Theorem discussed above, it can be seen that these rectangles strengthened by walls cannot have a lower collapse load than the full rectangular section.

Reference {1} contains a conservative formula for the lower bound collapse load of a clamped rectangular plate subject to a uniform pressure. The unsupported sections of plate are best modelled as clamped because the rotation at the support is zero.

This formula, implemented in CIRCPLAT, is:

$$P = (16M_o/b^2)(1+S^2)$$

where a=plate length

b=plate breadth

S=b/a

This equation is a lower bound associated with the Johansen yield curve.

PROGRAM DESCRIPTION AND USE

CIRCPLAT was written using FORTRAN-77 programming language on a VAX-11 computer with the VAX/VMS operating system. The program was designed to be run interactively with a minimum of required information from the program user, while providing the user with the choice of inputting a greater amount of detailed information if it is considered necessary.

CIRCPLAT was designed to handle a maximum of twenty stiffeners (horizontal plus vertical) but this capacity could be increased by modifying the DIMENSION statement at the beginning of the program. Similarly, CIRCPLAT can accommodate up to a maximum of 12 stiffeners in any one direction (horizontal or vertical).

The stiffener spacing is assumed by CIRCPLAT to be uniform along the horizontal and vertical diameters. Thus,

when the user inputs the total number of vertical stiffeners, the total number of horizontal stiffeners, and the bulkhead diameter, the program computes the x and y coordinates of every beam intersection, including the coordinates of beam-boundary intersections. These coordinates are used to determine the span lengths that are needed for calculation of the local shear forces in the nodal equilibrium equations.

CIRCPLAT takes advantage of the symmetry that exists about the horizontal and vertical diameters and reduces the remaining input required (and the calculations) to only those concerned with the upper left quadrant of the circle (which includes any beam(s) that lie directly on the horizontal and/or vertical diameters). During user input, the horizontal beams are numbered 1,2,3,etc., starting from the top, counting down to (and including) the horizontal beam at the centerline (if there is one). Similarly, vertical beams are numbered 1,2,3,etc. from left to right.

Inputs for the material yield stress, plate thickness, then the stiffener scantlings are requested by the program and the plastic yield moments are calculated. As a convenience, the plastic moment resulting from the first set

of vertical stiffener scantlings is assigned to all vertical stiffeners, the user can then modify this value, stiffener by stiffener, as required. The user has the option of revising the plastic moment of each stiffener by either

1. Inputting a full set of revised scantlings for that stiffener from which the program calculates the plastic yield moment; or

2. Inputting the plastic yield moment value directly for the specified beam.

Once the vertical beams have been fully specified, the identical routine is run for the horizontal beams.

The option of specifying the plastic yield moment value directly for an individual beam was added to allow the user to insert a calculated plastic moment corrected for the effect of shear stress or axial force if it should become necessary. The method of obtaining these corrections is discussed in detail in reference {4}.

CIRCPLAT was designed for analysis of a grillage where all beams are prismatic. For those cases involving beams

that vary in cross-section along their length, the minimum plastic yield moment should be used to obtain a conservative failure load, or, the program itself will have to be modified to allow for changes in stiffener geometry along a given beam length.

For those beams where shear deformations or effects must be taken into account, if the corrected plastic yield moment is less than the minimum plastic yield moment for the uncorrected beam, the corrected value should be input. Otherwise, to remain conservative, the minimum uncorrected plastic yield moment should continue to be used.

The current version of CIRCPLAT is designed to analyze the built-in submersible bulkhead. The program treats the grillage as being fully clamped at all boundaries.

At this point, the user input is completed, and the program begins to calculate the parameters that it will have to input to the ZX3LP linear programming subroutine. Basically, the remainder of the program is designed to accomplish four functions:

1. Loading the correct values into the proper locations in Matrices A,B,and C as indicated in the example of Chapter Three.

2. Establishing the exact matrix DIMENSION values required by ZX3LP.

3. Calculating the failure load of the unsupported sections of plating between stiffeners.

4. Formatting the solution produced by ZX3LP to produce a properly labelled output.

The solution is stored automatically in a data file labelled FOR018.DAT. This data file contains a listing of the input data and the output data.The output data consists of a listing of the following items:

1. The failure load of the weakest segment of unsupported plating.

2. The failure load of the grillage as a whole. This value is unrelated to the value in (1) above (i.e. they are treated as separate problems).

3. The normalized value (with the corresponding plastic moment value used to normalize it) of the moment present at failure at each of the vertical beam nodes. This is followed by a similar listing for all horizontal nodes. The listing of node moments is only for those nodes in the upper left quadrant of the plate. The rest are symmetric. Also, the vertical nodes are numbered from the top of the beam downward, working from left to right. In a similar fashion, the horizontal nodes are listed from left to right, working from top to bottom.

CHAPTER 5 RESULTS AND DISCUSSION

COMPARISON OF RESULTS

The results of CIRCPLAT were compared to the results obtained from an actual scale model failure test of a stiffened circular bulkhead (Reference {16}). The ratio of CIRCPLAT predicted collapse pressure to experimental collapse pressure was 0.61. There are a number of valid reasons why the CIRCPLAT prediction was 39% lower than the experimental value.

First, the horizontal and vertical stiffeners on the bulkhead under test were not prismatic. That is, the scantlings varied significantly along the stiffener length. To accommodate possibly large shear effects on the main horizontal girder, the web depth increased in a linear fashion (over approximately one quater span) at each end to a web depth almost twice that of the center span. Variations of web depth of a similiar magnitude were also common on vertical stiffeners. Because the program assumes

prismatic beams, and to ensure conservative results, the smallest value for web depth was input for every stiffener.

Secondly, the bulkhead design tested incorporated both HY-80 and HY-130 steels. Again, because the program allowed for only one material yield strength, and to ensure conservative results only the lower yield stress was used in the analysis.

Finally, a less important factor, the program can presently handle a maximum of twelve stiffeners in any one direction. There are twelve tertiary horizontal stiffeners and one main horizontal girder for the tested bulkhead (for a total of thirteen). To maintain the odd-symmetry relationship, and keep the total under twelve, it was necessary to reduce the number of tertiary stiffeners to ten. This has a minor effect, as indicated by the fact that the effect of tertiary stiffeners was ignored entirely in the analysis of reference {4}. Again, the results will be more conservative.

As a result of the above three factors, CIRCPLAT results can be assumed to be very conservative; this is borne out by the above comparison.

The previous results were for a strengthened version of an existing circular bulkhead design. For the original (not strengthened) version of this bulkhead, scantling data was readily available, but the given collapse load data was based on design values, rather than actual failure testing. CIRCPLAT was run and results were compared to the design collapse load for the circular bulkhead of the original design. When CIRCPLAT was run with tertiary stiffeners not included, the ratio of CIRCPLAT collapse pressure to design collapse pressure was 0.83. When CIRCPLAT was run with all tertiary stiffeners (8 of them) included, the ratio was 1.27. The correlation here is considered good, and the effect of tertiary stiffeners for this case would seem to be significant. Failure load analysis of the unsupported plating segments for the original design indicated that the unsupported plating was the weakest portion. The ratio of theoretical collapse pressure to design collapse pressure for the unsupported plate (clamped rectangular plate) was 0.73 for an Upper Bound solution, and 0.49 for a Lower Bound solution. This indicated the bulkhead plating would not withstand full design pressure. This problem did not show up in the improved bulkhead design.

In reference {10}, Hodge provides a formula for the exact collapse load of a uniform circular plate, with a fully clamped boundary. The formula reduces to

$$P = 2.814\sigma_o t^2/R^2$$

where P = uniform collapse pressure

σ_o = material yield stress

t = plate thickness,

R = Radius of plate.

For a plate 10 feet in diameter, 2 inches thick, with a yield stress of 50,000 psi, the exact uniform collapse pressure is 156 psi.

Referring to the formulas of Chapter 3 for calculating the plastic section modulus of a stiffened plate, CIRCPLAT can be used to accurately model a circular plate as a grillage by setting all web and flange scantlings equal to zero, except for the web depth. The web depth can be any arbitrary value, as it factors out of the plastic section modulus formula, but it must not be zero, because it is in the denominator of r_3 and r_4 and would cause these values to become singular. For convenience, input web depth to CIRCPLAT as 1.0, and input all other web and flange scantlings as zero. As a result, the plate will be modelled

by CIRCPLAT as a grillage of stiffeners with rectangular cross section (bxh) where b is stiffener spacing, determined by the number of stiffeners the user inputs (more stiffeners will give better results); and h is plating thickness.

For the same plate analyzed above using Hodges's formula, CIRCPLAT calculated a failure load of 154.7 psi, only 1% less than the exact value. The plate was modelled for this case as ten vertical and ten horizontal stiffeners. This result is significant in that it points out that CIRCPLAT does yield a conservative yet very close, lower bound solution, in a case where the input values can closely approximate the exact physical problem, and also verifies that a uniform load can be approximated quite well by point loads at all nodes.

SUGGESTED IMPROVEMENTS TO CIRCPLAT

In order to reduce the amount of conservatism indicated

by the comparison of CIRCPLAT results to actual stiffened plate failure test data, several improvements to the program are recommended for implementation:

1. Modify the input routine to provide the user with the ability to input a different set of horizontal and/or vertical stiffener scantlings at each nodal point. This would enable CIRCPLAT to more closely approximate a non-prismatic beam;

2. Modify the input routine to provide the user with the ability to input a different yield stress value for each stiffener, or for the circular plate. This would, again, provide the user with more flexibility in modelling a real, physical problem;

3. Modify the program to include a variable "fixity" factor, f, at boundary nodes, ranging in value from zero to one. This would allow the user to model a simple support (f=0.), a fixed support (f=1.), or some value in between.

CONCLUSION

In conclusion, it is felt that the linear programming method of grillage analysis, based on the Lower Bound Theorem, provides a simple, yet powerful tool for accurately determining the collapse load of a given structure. CIRCPLAT was designed for the problem of the circular plate stiffened by a grillage network, with fixed (clamped) boundaries. However, the applicability of this technique can be extended very easily to all types of grillage collapse analysis. It is strongly felt that this analysis tool yields results that are consistent with empirical data.

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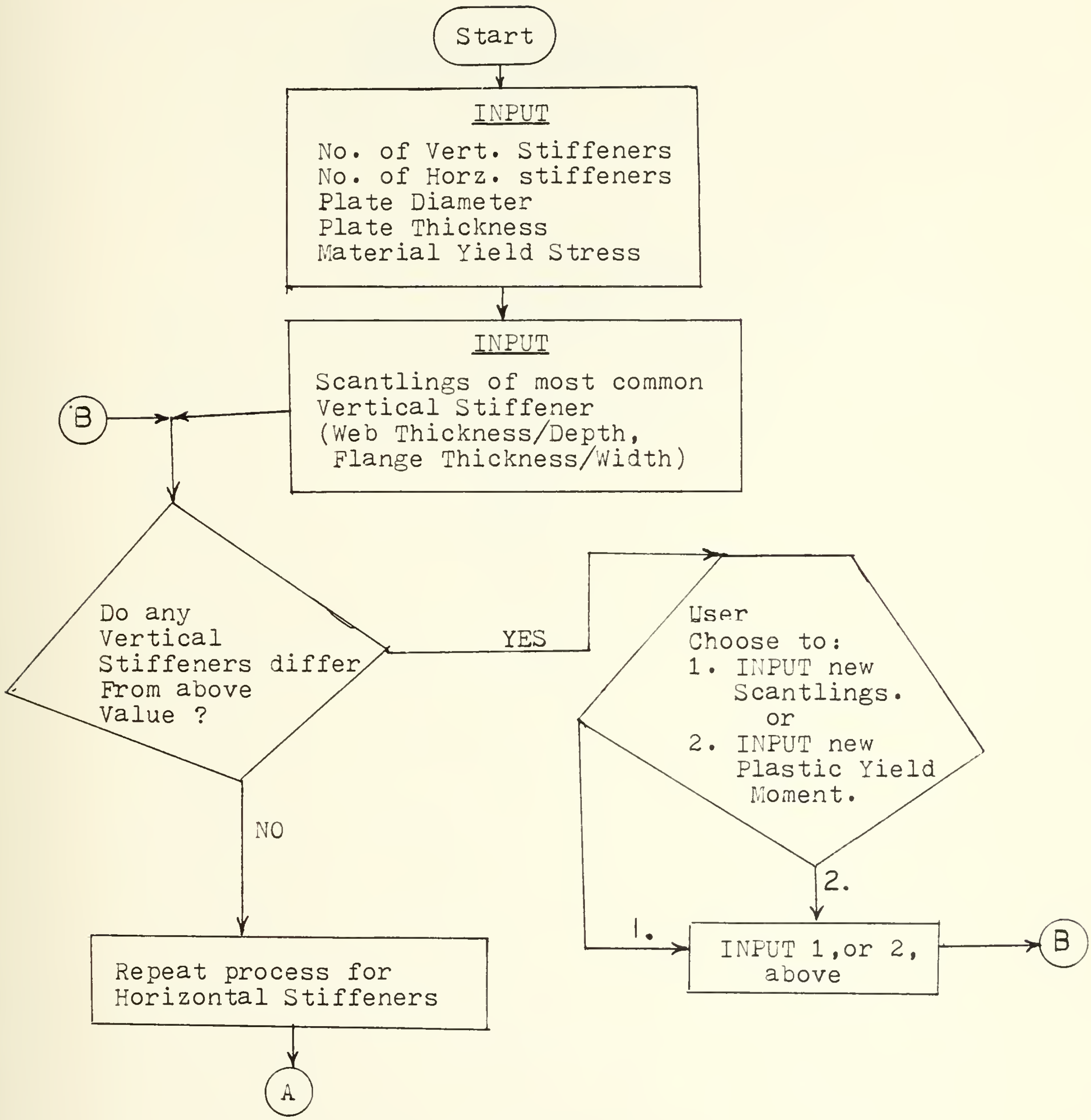
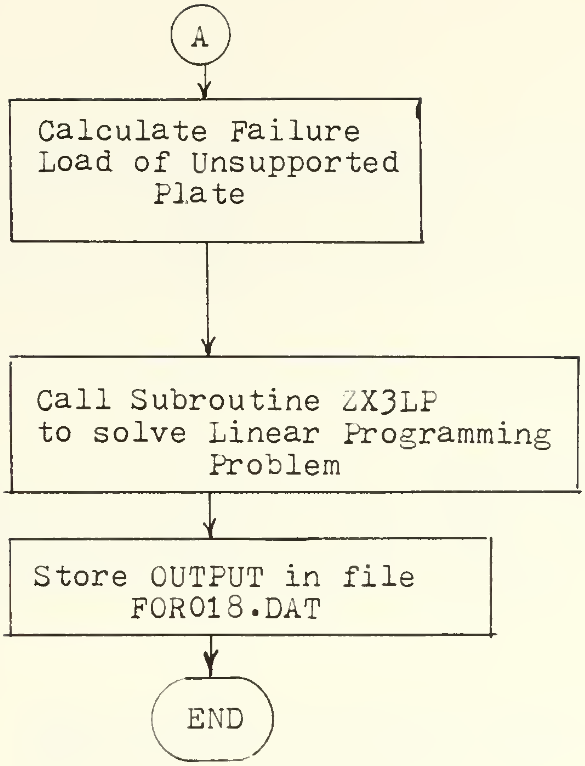

Start
INPUT
No. of Vert. Stiffeners
No. of Horz. stiffeners
Plate Diameter
Plate Thickness
Material Yield Stress
INPUT
Scantlings of most common
Vertical Stiffener
(Web Thickness/Depth,
Flange Thickness/Width)
B
Do any
Vertical
Stiffeners differ
From above
Value ?
YES
User
Choose to:
1. INPUT new
Scantlings.
or
2. INPUT new
Plastic Yield
Moment.
2.
1.
INPUT 1,or 2,
above
B
NO
Repeat process for
Horizontal Stiffeners
A


A
Calculate Failure Load of Unsupported Plate
Call Subroutine ZX3LP to solve Linear Programming Problem
Store OUTPUT in file FOR018.DAT
END

```
C23456789
      DIMENSION XCOORD(8,8),YCOORD(8,8),VERTMO(8),
     1 HORZMO(8),MV(8,8),MH(8,8),NNODE(40),F(40),
     1 NVERT(40),MLIN(80),MMAX(80),C(80),AA(120,80),
     1 B(100),HORZL(40),VERTL(40),KHORZ(40),KVERT(40),
     1 MPV(45),MPH(45),NVLEN(45),IFLAG(45),PSOL(102),
     1 A(120,80),DSOL(120),RW(8000),IW(304)

      REAL HORZMO,MV,MH,MMAX,MLIN,HORZL,MPV,MPH,HFLAG,MP

C
C   USER INTERACTIVELY INPUTS BEAM AND PLATE PARAMETERS FOR
C   THE STIFFENED CIRCULAR PLATE OF INTEREST. SYMMETRY ABOUT
C   THE HORIZONTAL AND VERTICAL CENTERLINES IS ASSUMED.
C   INPUT PARAMETERS AND CALCULATIONS, THEN, ARE FOR THE
C   UPPER LEFT QUADRANT OF THE CIRCULAR PLATE.
C

C234567
      PRINT *, '  THIS PROGRAM WAS DESIGNED FOR A LIMIT OF'
      PRINT *,'  TWENTY TOTAL STIFFENERS (HORIZ.+VERT.)'
      PRINT *,'  TWELVE MAX. IN ONE DIRECTION.'
      Y=SKIP(5)
      PRINT *, '  THIS PROGRAM ASSUMES THAT HORIZONTAL AND'
      PRINT *, '  VERTICAL STIFFENERS ARE EQUALLY SPACED ALONG'
      PRINT *, '  THE HORIZONTAL AND VERTICAL CENTERLINES, AND'
      PRINT *, '  ARE T-STIFFENERS WITH AN EFFECTIVE PLATE WIDTH'
      PRINT *, '  MAKING THEM I-STIFFENERS.'
      Y=SKIP(5)
      PRINT *, '  ENTER THE NO. OF VERTICAL STIFFENERS'
      READ *, N2
      Y=SKIP(5)
      PRINT *, '  ENTER THE NO. OF HORIZONTAL STIFFENERS'
      READ *, M2
      Y=SKIP(5)
      PRINT *, '  ENTER THE DIAMETER (FEET) OF THE CIRC. PLATE'
      READ *, DIAM
      D=DIAM*12.
      R=D/2.
      WRITE(18,4000)
 4000 FORMAT(1X,'INPUT INFORMATION'/)
C234567
      WRITE(18,4001) N2
 4001 FORMAT(1X,'NO. OF VERTICAL STIFFENERS:',3X,I3 /)
      WRITE(18,4002) M2
 4002 FORMAT(1X,'NO. OF HORIZONTAL STIFFENERS:',3X,I3 /)
      WRITE(18,4003) DIAM
```

```
 4003 FORMAT(1X,'PLATE DIAMETER:',3X,F6.2,'(FEET)' /)
C
C  CALCULATE THE X (HORIZ.) AND Y (VERT.) COORDINATES OF
C  NODES (BEAM INTERSECTIONS OR BOUNDARIES)
C
C
      YDIST=D/REAL(M2+1)
      XDIST=D/REAL(N2+1)
C CHECK ODD OR EVEN SYMMETRY
      VAR1=REAL(N2)/2.-IFIX(REAL(N2)/2.)-.1
      VAR2=REAL(M2)/2.-IFIX(REAL(M2)/2.)-.1
      IODDV=0
      IODDH=0
      IF(VAR1.GT.0) IODDV=1
      IF(VAR2.GT.0) IODDH=1
      IF(IODDV.EQ.0) GO TO 50
      KCOL=(N2+1)/2+1
      VFLAG=0.0
      GO TO 51
 50   KCOL=N2/2+1
      VFLAG=1.0
 51   IF(IODDH.EQ.0) GO TO 55
      KROW=(M2+1)/2+1
      HFLAG=0.0
      GO TO 56
 55   KROW=M2/2+1
      HFLAG=1.0
 56   CONTINUE
C
C  KCOL = NO. OF NODES ON LONGEST HORIZ. BEAM
C  KROW = NO. OF NODES ON LONGEST VERT. BEAM
C  (THIS IS INCLUDING BOUNDARY NODES AND IS FOR
C  THE UPPER LEFT QUADRANT ONLY)
C  H/VFLAG = 0 IF ODD SYMMETRY
C          = 1 IF EVEN SYMMETRY
C
C234
C  INITIALIZE ALL COORD. MATRICES TO ZERO
C
      DO 70 I=1,KROW
         DO 60 J=1,KCOL
            XCOORD(I,J)=0.0
            YCOORD(I,J)=0.0
 60      CONTINUE
 70   CONTINUE
C
C  FILL IN COORD. MATRICES
C
```

```
         DO 110 NROW=1,KROW
            DO 100 NCOL=KCOL,1,-1
               IF(NROW.GT.1) GO TO 115
               XVAL=R-(REAL(KCOL-NCOL)+VFLAG/2.)*XDIST
               YVAL=SQRT(R*R-(R-XVAL)**2)
               YMIN=(REAL(KROW)-2+HFLAG/2.)*YDIST
               TEST1=YVAL-YMIN
               IF(TEST1.LT.0) GO TO 110
               XCOORD(NROW,NCOL)=XVAL
               YCOORD(NROW,NCOL)=YVAL
               GO TO 100
  115          XMAX=SQRT(R*R-((REAL(KROW-NROW)+HFLAG/2.)*YDIST)**2)
               YMAX=SQRT(R*R-((REAL(KCOL-NCOL)+VFLAG/2.)*XDIST)**2)
C234567
               IF(NCOL.EQ.1) GO TO 116
               XVAL=R-(REAL(KCOL-NCOL)+VFLAG/2.)*XDIST
               XCOORD(NROW,NCOL)=XVAL
               TEST2=XMAX-(R-XVAL)
               IF(TEST2.LE.0) GO TO 105
  106          YVAL=(REAL(KROW-NROW)+HFLAG/2.)*YDIST
               TEST3=YMAX-YVAL
               IF(TEST3.LT.-YDIST) GO TO 110
               IF(TEST3.LE.0) GO TO 108
               YCOORD(NROW,NCOL)=YVAL
              GO TO 100
  105         XCOORD(NROW,NCOL)=R-XMAX
              GO TO 106
  108         YCOORD(NROW,NCOL)=YMAX
              GO TO 110
  116         XCOORD(NROW,NCOL)=R-XMAX
              YCOORD(NROW,NCOL)=(REAL(KROW-NROW)+HFLAG/2.)*YDIST
              GO TO 110
  100       CONTINUE
  110    CONTINUE
C
C  USER INPUTS STIFFENER SCANTLINGS TO DETERMINE PLASTIC MOMENTS
C
      PRINT *, '  INPUT MATERIAL YIELD STRESS (PSI) AND PLATE'
      PRINT *, '  THICKNESS (INCHES) AS FOLLOWS:'
      PRINT *, '  SIGMA-Y, THICKNESS'
      READ *, SIGMAY,T2
      Y=SKIP(5)
      PRINT *, '  THE INITIAL SCANTLINGS YOU INPUT WILL BE ASSIGNED'
      PRINT *, '  TO ALL VERTICAL STIFFENERS. YOU WILL THEN HAVE THE
      PRINT *, '  OPTION OF REVISING SPECIFIC STIFFENERS BY NUMBER.'
      PRINT *, '  (BY ENTERING EITHER SCANTLINGS OR PLASTIC MOMENT)'
      Y=SKIP(5)
      IVERT=0
```

```
      PRINT *, '  INPUT INITIAL VERTICAL BEAM SCANTLINGS AS FOLLOWS:
15    PRINT *, ' ENTER WEB THICKNESS,WEB DEPTH, BOTTOM FLANGE '
      PRINT *, ' THICKNESS,BOTTOM FLANGE WIDTH (ALL IN INCHES)'
      READ *, T3,DW,T1,B1
      IF(IVERT.EQ.0) GO TO 4005
      WRITE(18,4006) NBEAM-1,T3,DW,T1,B1
4006  FORMAT(1X,'VERTICAL BEAM NO.:',3X,I3,/
     11X,'WEB THICKNESS:',3X,F8.3,' INCHES',/
     11X,'WEB   DEPTH  :',3X,F8.3,' INCHES',/
     11X,'BOTTOM FLANGE THICKNESS:',3X,F8.3,' INCHES',/
     11X,'BOTTOM FLANGE   WIDTH  :',3X,F8.3,' INCHES',//)
      GO TO 16
4005  WRITE(18,4004) SIGMAY,T2,T3,DW,T1,B1
4004  FORMAT(1X,' VERTICAL BEAMS ',/
     11X,'MATERIAL YIELD STRESS:',3X,F12.2,' PSI',/
     11X,'PLATE THICKNESS:',3X,F8.3,' INCHES',/
     11X,'COMMON WEB THICKNESS:',3X,F8.3,' INCHES',/
     11X,'COMMON WEB DEPTH    :',3X,F8.3,' INCHES',/
     11X,'COMMON BOTTOM FLANGE THICKNESS:',3X,F8.3,' INCHES',/
     11X,'COMMON BOTTOM FLANGE WIDTH    :',3X,F8.3,' INCHES',//)
   16 IF(IVERT.EQ.1) GO TO 21
      DO 20 I=2,KCOL
         VERTMO(I)=MP(DW,XDIST,T2,B1,T1,SIGMAY,T3)
20    CONTINUE
      GO TO 22
21    VERTMO(NBEAM)=MP(DW,XDIST,T2,B1,T1,SIGMAY,T3)
22    PRINT *, '  ARE THERE ANY VERT. BEAMS WHICH ARE DIFFERENT'
      PRINT *, '  FROM THOSE INPUT ABOVE ?'
      PRINT *, '  0=NO, 1=YES'
      READ *, IVERT
      IF(IVERT.EQ.0) GO TO 30
      Y=SKIP(2)
      PRINT *, '  SINCE SYMMETRY IS ASSUMED, INPUT ONLY THOSE VERT.'
      PRINT *, '  BEAMS ON OR TO THE LEFT OF VERT. CENTERLINE.'
      Y=SKIP(2)
      PRINT *, '  COUNTING IN SEQUENCE LEFT TO RIGHT, INPUT THE'
      PRINT *, '  BEAM NO. FOR WHICH THERE ARE NEW SCANTLINGS'
      READ *, NBEAM
      NBEAM=NBEAM+1
      PRINT *, '  DO YOU WANT TO INPUT SCANTLINGS, OR PLASTIC MOMENT?
      PRINT *, '  0=INPUT PLASTIC MOMENT,1=INPUT SCANTLINGS'
      READ *, NOPT
      IF(NOPT.EQ.1) GO TO 15
      PRINT*, '  INPUT PLASTIC MOMENT'
      READ *, VERTMO(NBEAM)
      Y=SKIP(2)
      GO TO 22
30    IHORZ=0
```

```
      PRINT *, '  NOW A SIMILAR ROUTINE WILL BE FOLLOWED FOR'
      PRINT *, '  HORIZONTAL STIFFENERS.'
      PRINT *, '  INPUT HORIZ. BEAM SCANTLINGS AS FOLLOWS:'
35    PRINT *, '  ENTER WEB THICKNESS, WEB DEPTH, BOTTOM FLANGE'
      PRINT *, '  THICKNESS, BOTTOM FLANGE WIDTH (ALL IN INCHES)'
      READ *, T3,DW,T1,B1
      IF(IHORZ.EQ.0) GO TO 4007
      WRITE(18,4008) MBEAM-1,T3,DW,T1,B1
4008  FORMAT(1X,'HORIZONTAL BEAM NO.:',3X,I3,/
     11X,'WEB THICKNESS:',3X,F8.3,' INCHES',/
     11X,'WEB   DEPTH  :',3X,F8.3,' INCHES',/
     11X,'BOTTOM FLANGE THICKNESS:',3X,F8.3,' INCHES',/
     11X,'BOTTOM FLANGE   DEPTH  :',3X,F8.3,' INCHES',//)
      GO TO 34
4007  WRITE(18,4009) T3,DW,T1,B1
4009  FORMAT(1X,'HORIZONTAL BEAMS',/
     11X,'COMMON WEB THICKNESS:',3X,F8.3,' INCHES',/
     11X,'COMMON WEB   DEPTH  :',3X,F8.3,' INCHES',/
     11X,'COMMON BOTT.FLG.THICKNESS:',3X,F8.3,' INCHES',/
     11X,'COMMON BOTT.FLG.  WIDTH  :',3X,F8.3,' INCHES',//)
34    IF(IHORZ.EQ.1) GO TO 31
      DO 33 I=2,KROW
         HORZMO(I)=MP(DW,YDIST,T2,B1,T1,SIGMAY,T3)
33    CONTINUE
      GO TO 32
31    HORZMO(MBEAM)=MP(DW,YDIST,T2,B1,T1,SIGMAY,T3)
32    PRINT *, '  ARE THERE ANY HORIZ. BEAMS WHICH ARE '
      PRINT *, '  DIFFERENT FROM THOSE INPUT ABOVE ?'
      PRINT *, '  0=NO, 1=YES'

      READ *, IHORZ
      IF(IHORZ.EQ.0) GO TO 40
      Y=SKIP(2)
C234567
      PRINT *, '  SINCE SYMMETRY ASSUMED, INPUT ONLY THOSE'
      PRINT *, '  HORIZ. BEAMS ON OR ABOVE HORIZ. CENTERLINE'
      Y=SKIP(2)
      PRINT *, '  COUNTING IN SEQUENCE FROM TOP TO BOTTOM,'
      PRINT *, '  INPUT THE BEAM NO. FOR THE NEW SCANTLINGS.'
      READ *, MBEAM
      MBEAM=MBEAM+1
      PRINT *, '  DO YOU WANT TO INPUT SCANTLINGS OR PLASTIC MOMENT?'
      PRINT *, '  0=INPUT PLASTIC MOMENT, 1=INPUT SCANTLINGS'
      READ *, MOPT
      IF(MOPT.EQ.1) GO TO 35
      PRINT *, '  INPUT PLASTIC MOMENT'
      READ *, HORZMO(MBEAM)
      Y=SKIP(2)
```

```
      GO TO 32
 40   CONTINUE
C
C
C  INITIALIZE ALL MV AND MH FIELDS TO ZERO
C
C
      DO 90 I=1,KROW
          DO 80 J=1,KCOL
              MV(I,J)=0.0
              MH(I,J)=0.0
 80       CONTINUE
 90   CONTINUE
C234567
C
C
C  INITIALIZE NODAL EQUATION COUNTER TO ZERO AND COUNT
C  INTERNAL NODES FOR EACH LINE. SUM INTERNAL NODES TO
C  COUNT NODAL EQUATIONS
C
C
      NODEEQ=0
      DO 120 J=2,KROW
          XMAX=SQRT(R*R-((REAL(KROW-J)+HFLAG/2.)*YDIST)**2)
          NNODE(J)=IFIX(XMAX/XDIST-VFLAG/2.+0.9999)
          NODEEQ=NODEEQ+NNODE(J)
 120  CONTINUE
C
C
C  CREATE A ONE DIMENSIONAL ARRAY OF UNIT NODAL GRID LOADS
C
C
      DO 130 K=1,NODEEQ
         F(K)=1
 130  CONTINUE
      KKK=1
      IF(KKK.GT.0) IBC=2
      IF(KKK.GT.0) GO TO 131
      PRINT *, '  ENTER BOUNDARY CONDITIONS:'
      PRINT *, '  1=SIMPLE SUPPORT, 2=FIXED SUPPORT'
      READ *, IBC
 131  CONTINUE
C
C
C   END OF USER INPUT
C
      WRITE(18,4014)
 4014 FORMAT(1X,' OUTPUT INFORMATION ')
```

```
      IF(NOPT.EQ.1.AND.MOPT.EQ.1.AND.T3.LT.0.00001) GO TO 4020
      BB=YDIST/2.
      L=XDIST/2.
      IF(XDIST.GE.YDIST) BB=XDIST/2.
      IF(XDIST.GE.YDIST) L=YDIST/2.
      PL=SIGMAY*(T2**2/BB**2)*(1+BB**2/L**2)
      WRITE(18,4015) PL
 4015 FORMAT(1X,' UNSUPP. PLATE SECTION COLLAPSE LOAD:',F10.4)
C
C
C  ASSIGN MOMENT FLAGS TO HORIZ. BOUNDARY NODES IAW INPUT B.C.
C
C
C
C234567
 4020 DO 140 NN=2,KROW
          MCOL=KCOL-NNODE(NN)
          MH(NN,MCOL)=1.
          IF(IBC.EQ.1) MH(NN,MCOL)=0.0
 140  CONTINUE
C
C  COUNT INTERNAL NODES ON VERTICAL BEAMS
C
      DO 150 L=2,KCOL
          YMAX=SQRT(R*R-((REAL(KCOL-L)+VFLAG/2.)*XDIST)**2)
          NVERT(L)=IFIX(YMAX/YDIST-HFLAG/2.+0.9999)
 150  CONTINUE
C234567
C
C  ASSIGN MOMENT FLAGS TO VERT. BOUNDARY NODES IAW INPUT B.C.
C
C
      DO 160 JK=2,KCOL
          MROW=KROW-NVERT(JK)
          MV(MROW,JK)=1.
          IF(IBC.EQ.1) MV(MROW,JK)=0.0
 160  CONTINUE
C
C  SET INTERNAL NODE MOMENT FLAGS
C
      DO 180 JL=2,KROW
         NSTART=KCOL-NNODE(JL)+1
         DO 170 JN=NSTART,KCOL
            MV(JL,JN)=1.
            MH(JL,JN)=1.
 170     CONTINUE
 180  CONTINUE
C234567
```

```
C
C  CREATE A LINE ARRAY OF NON-ZERO VERTICAL MOMENTS AND AN
C  ASSOCIATED ARRAY OF MAXIMUM PLASTIC MOMENTS; COUNT
C  TOP TO BOTTOM, LEFT TO RIGHT
C
      NLV=1
      DO 210 J=2,KCOL
         DO 200 I=1,KROW
             IF(MV(I,J).LT.1) GO TO 200
             MLIN(NLV)=MV(I,J)
             MMAX(NLV)=VERTMO(J)
             NLV=NLV+1
 200     CONTINUE
 210  CONTINUE
C
C  CONTINUE THE LINE ARRAY WITH NON-ZERO HORIZ. MOMENTS AND THEIR
C  ASSOCIATED LIMITS.
C
C234567
      NLH=NLV
      DO 230 I=2,KROW
         DO 220 J=1,KCOL
            IF(MH(I,J).LT.1) GO TO 220
            MLIN(NLH)=MH(I,J)
            MMAX(NLH)=HORZMO(I)
            NLH=NLH+1
 220     CONTINUE
 230     CONTINUE
C
C
C  CALCULATE TOTAL NUMBER OF EQUATIONS (NODAL EQUILIBRIUM PLUS
C  INEQUALITY EQUATIONS)
      NEQTOT=NODEEQ+NLH-1
C
C
C  ESTABLISH THE C MATRIX (OBJECTIVE FUNCTION VECTOR)
C  THIS IS THE FINAL C MATRIX
C
      DO 240 I=1,NLH
         C(I)=0.0
 240  CONTINUE
      C(NLH)=1.0
C
C
C  INITIALLIZE THE AA MATRIX (VARIABLE COEFFICIENTS)
C  NOTE: NO. OF ROWS REQUIRED BY ZX3LP IS NEQTOT+2
C
C
```

```
      DO 250 I=1,NEQTOT+2
         DO 245 J=1,NLH
            AA(I,J)=0.0
 245     CONTINUE
 250  CONTINUE
C234567
C
C  INITIALIZE THE B MATRIX (RIGHT HAND SIDE VECTOR)
C  FILL IN ANY KNOWN (INEQUALITY) VALUES FOR B
C  IF BOUNDARIES ARE FIXED, THIS IS THE FINAL B MATRIX
C
      DO 255 I=1,NEQTOT+2
         IF(I.GT.NLH-1) GO TO 254
         B(I)=2.*MMAX(I)
         GO TO 255
 254     B(I)=0.0
 255  CONTINUE
C
C  FILL IN AA MATRIX INEQUALITY CONSTRAINT VARIABLE COEFFICIENTS
C
      DO 265 I=1,NLH-1
        AA(I,I)=1.
 265  CONTINUE
C
C  LOAD NODAL FORCE COEFFICIENTS INTO THE AA MATRIX
C
      INDEX=NLH
      DO  270 I=1,NODEEQ
         AA(INDEX,NLH)= -1.0
         INDEX=INDEX+1
 270  CONTINUE
C
C  CREATE A LINE ARRAY OF HORIZ. LENGTHS, AND ANOTHER ARRAY
C  OF VERTICAL LENGTHS
C
C    HORIZONTAL ARRAY:
C
      NNN=1
      DO 280 J=2,KROW
        NSTART=KCOL-NNODE(J)+1
        DO 275 L=NSTART,KCOL
           HORZL(NNN)=XCOORD(J,L)-XCOORD(J,L-1)
           NNN=NNN+1
 275    CONTINUE
 280  CONTINUE
C
C  VERTICAL ARRAY:
```

```
      NNN=1
      DO 290 J=2,KCOL
         NSTART=KROW-NVERT(J)+1
         DO 285 I=NSTART,KROW
            VERTL(NNN)=YCOORD(I-1,J)-YCOORD(I,J)
            NNN=NNN+1
 285     CONTINUE
 290  CONTINUE
C
C  CREATE AN ARRAY (KHORZ(N)//KVERT(N)) WHICH ASSOCIATES THE
C  SEQUENTIAL (HORIZONTAL//VERTICAL) NODE NUMBER TO EACH INTERNAL
C  NODE NUMBER. INTERNAL NODE NUMBERING LEFT-RIGHT,TOP-BOTTOM
C
      NINT=1
      DO 300 I=2,KROW
         NSTART=KCOL-NNODE(I)+1
         DO 295 J=NSTART,KCOL
             IF(IBC.EQ.1) KHORZ(NINT)=NINT
             IF(IBC.EQ.2) KHORZ(NINT)=NINT+(I-1)
C234567890123456789012345678901234567890123456789012345678901234567890
             IF(J.EQ.2) KOUNT=0
             IF(J.GT.2) GO TO 296
 298         KVERT(NINT)=KOUNT+I+IBC-1-(KROW-NVERT(J))
             GO TO 299
 296         KOUNT=0
             DO 297 K=2,J-1
                 KOUNT=KOUNT+NVERT(K)+IBC-1
 297         CONTINUE
             GO TO 298
 299         NINT=NINT+1
 295     CONTINUE
 300  CONTINUE
C
C  CREATE A LINEAR ARRAY OF HORIZ. AND VERT. PLASTIC MOMENTS
C  ASSOCIATED WITH EACH INTERNAL NODE
C
      NINT=1
      DO 310 I=2,KROW
         NSTART=KCOL-NNODE(I)+1
         DO 305 J=NSTART,KCOL
            MPV(NINT)=VERTMO(J)
            MPH(NINT)=HORZMO(I)
            NINT=NINT+1
 305     CONTINUE
 310  CONTINUE
C
C  CREATE AN ARRAY ASSOCIATING VERTICAL LENGTH INDICES WITH
C  INTERNAL NODES
```

```
C
      NINT=1
      DO 330 I=2,KROW
         NSTART=KCOL-NNODE(I)+1
         DO 325 J=NSTART,KCOL
            IF(J.EQ.2) KOUNT=0
            IF(J.GT.2) GO TO 326
 328        NVLEN(NINT)=KOUNT+I-(KROW-NVERT(J))
            GO TO 329
 326        KOUNT=0
            DO 327 K=2,J-1
               KOUNT=KOUNT+NVERT(K)
 327        CONTINUE
            GO TO 328
 329        NINT=NINT+1
 325     CONTINUE
 330  CONTINUE
C
C    CREATE AN ARRAY OF FLAGS ASSOCIATED WITH EACH INTERNAL NODE
C    TO INDICATE THE FOLLOWING FLAG CONDITION:
C       (1) NODE IS NEAR VERTICAL BOUNDARY
C       (2) NODE IS NEAR HORIZONTAL BOUNDARY
C       (3) NODE IS NEAR BOTH BOUNDARIES
C       (4) NODE IS A SYMMETRY  NODE ABOUT THE HORIZ. DIAMETER
C       (5) NODE IS A  SYMMETRY NODE ABOUT THE VERT. DIAMETER
C       (6) NODE IS NEAR BOUNDARY AND NEAR HORIZONTAL AXIS OF SYMMETRY
C       (7) NODE IS NEAR BOUNDARY AND NEAR VERTICAL AXIS OF SYMMETRY
C
      NINT=1
      DO 320 I=2,KROW
         NSTART=KCOL-NNODE(I)+1
         DO 315 J=NSTART,KCOL
            IF(HORZL(NINT).LT.XDIST) IFLAG(NINT)=2
            IF(VERTL(NVLEN(NINT)).LT.YDIST) IFLAG(NINT)=1
            IF(VERTL(NVLEN(NINT)).LT.YDIST.AND.HORZL(NINT).LT.XDIST)
C234567890123456789012345678901234567890123456789012345678901234567890
     1  IFLAG(NINT)=3
            IF(J.EQ.KCOL) IFLAG(NINT)=5
            IF(J.EQ.KCOL.AND.I.EQ.2) IFLAG(NINT)=7
            IF(I.EQ.KROW) IFLAG(NINT)=4
            IF(I.EQ.KROW.AND.J.EQ.NSTART) IFLAG(NINT)=6
            NINT=NINT+1
 315     CONTINUE
 320  CONTINUE
      DO 1000 I=1,NODEEQ
         WRITE(6,1001) HORZL(I),NVLEN(I),VERTL(NVLEN(I))
 1000 CONTINUE
 1001 FORMAT(1X,F8.3,I8,F8.3)
```

```
C234567
C
C LOAD NODAL EQUATION COEFFICIENTS INTO AA MATRIX
C
      INDEX=NLH-1
      DO 340 I=1,NODEEQ
         IF(IBC.EQ.1) GO TO 341
         KHVAL=NLV-1+KHORZ(I)
         KVVAL=KVERT(I)
         IF(I.EQ.NODEEQ) GO TO 342
         AA(INDEX+I,KHVAL-1)= -1./HORZL(I)
         IF(IFLAG(I).EQ.5.AND.IODDV.EQ.1) GO TO 343
         IF(IFLAG(I).EQ.5.AND.IODDV.EQ.0) GO TO 344
         IF(IBC.EQ.2.AND.IFLAG(I).EQ.7.AND.IODDV.EQ.1) GO TO 343
         IF(IBC.EQ.2.AND.IFLAG(I).EQ.7.AND.IODDV.EQ.0) GO TO 344
         AA(INDEX+I,KHVAL)= 1./HORZL(I)+ 1./HORZL(I+1)
         AA(INDEX+I,KHVAL+1)= -1./HORZL(I+1)
         GO TO 345
 343     AA(INDEX+I,KHVAL-1)= -2./HORZL(I)
         AA(INDEX+I,KHVAL)= 2./HORZL(I)
         GO TO 345
 344     AA(INDEX+I,KHVAL-1)= -1./HORZL(I)
         AA(INDEX+I,KHVAL)= 1./HORZL(I)
         KVVAL=KVERT(I)
 345     IF(I.EQ.NODEEQ) GO TO 342
         AA(INDEX+I,KVVAL-1)= -1./VERTL(NVLEN(I))
         IF(IFLAG(I).EQ.4.AND.IODDH.EQ.1) GO TO 346
         IF(IFLAG(I).EQ.4.AND.IODDH.EQ.0) GO TO 347
         IF(IBC.EQ.2.AND.IFLAG(I).EQ.6.AND.IODDH.EQ.1) GO TO 346
         IF(IBC.EQ.2.AND.IFLAG(I).EQ.6.AND.IODDH.EQ.0) GO TO 347
         AA(INDEX+I,KVVAL)= 1./VERTL(NVLEN(I))+ 1./VERTL(NVLEN(I)+1)
C234567890123456789012345678901234567890123456789012345678901234567890
         AA(INDEX+I,KVVAL+1)= -1./VERTL(NVLEN(I)+1)
         GO TO 340
 346     AA(INDEX+I,KVVAL-1)= -2./VERTL(NVLEN(I))
         AA(INDEX+I,KVVAL)= 2./VERTL(NVLEN(I))
         GO TO 340
 347     AA(INDEX+I,KVVAL-1)= -1./VERTL(NVLEN(I))
         AA(INDEX+I,KVVAL)= 1./VERTL(NVLEN(I))
         GO TO 340
 342     IF(IODDV.EQ.1)GO TO 348
         AA(INDEX+I,KHVAL)= 1./HORZL(I)
         AA(INDEX+I,KHVAL-1)= -1./HORZL(I)
         GO TO 349
 348     AA(INDEX+I,KHVAL)= 2./HORZL(I)
         AA(INDEX+I,KHVAL-1)= -2./HORZL(I)
 349     IF(IODDH.EQ.0) GO TO 350
         AA(INDEX+I,KVVAL)= 1./VERTL(NVLEN(I))
```

```
              AA(INDEX+I,KVVAL-1)= -1./VERTL(NVLEN(I))
              GO TO 340
  350         AA(INDEX+I,KVVAL)= 2./VERTL(NVLEN(I))
              AA(INDEX+I,KVVAL-1)= -2./VERTL(NVLEN(I))
  340         CONTINUE
           IF(KKK.EQ.1) GO TO 341
              WRITE(18,3000)
              WRITE(18,3001) ((AA(I,J),J=1,NLH),I=1,NEQTOT+2)
  3000        FORMAT(1X,'THE AA MATRIX IS:')
  3001        FORMAT(1X,13(2X,F7.4))
  341      PRINT *, '  SIMPLE SUPPORT NOT FULLY IMPLEMENTED YET'
            IF(IBC.EQ.1) STOP
           M1=NLH-1
           M2=NODEEQ
           N=NLH
           IA=M2+M1+2
           IRW1=IA*IA+3*M1+2*M2+4
           IW1=2*M2+3*M1+4
           MN=MAX0(N,M1+M2)
           CALL DIMEN(A,AA,B,C,N,M1,M2,IA,IRW1,IW1,PSOL,DSOL,IW,RW,MN)
C
C CONVERT ZX3LP SOLUTION VECTOR PSOL(I) TO THE CORRESPONDING
C NORMALIZED MOMENTS AT THE NODES, AND ALSO DETERMINE THE
C APPLIED NODAL FORCE AND EQUIVALENT UNIFORM PRESSURE
C
           DO 400 I=1,NLH-1
               MLIN(I)=(PSOL(I)-MMAX(I))/MMAX(I)
  400      CONTINUE
           IF(HFLAG.EQ.1) HFAC=0
           IF(HFLAG.EQ.0) HFAC=1
           IF(VFLAG.EQ.1) IFAC=0
           IF(VFLAG.EQ.0) IFAC=1
           FNODE=PSOL(NLH)
           FTOT=FNODE*(4.*NODEEQ-HFAC*2.*NNODE(KROW)-IFAC*2.*NVERT(KCOL)
          1 +HFAC*IFAC)
           PRESS=FTOT/(3.14159265*R*R)
           WRITE(18,450) PRESS
           WRITE(18,460)
           WRITE(18,470)
           DO 500 I=1,NLV-1
                WRITE(18,480) I,MLIN(I),MMAX(I)
  500      CONTINUE
           WRITE(18,490)
           WRITE(18,485)
           DO 600 I=NLV,NLH-1
                WRITE(18,480) I-NLV+1,MLIN(I),MMAX(I)
  600      CONTINUE
  450      FORMAT(1X,'THE COLLAPSE PRESSURE IS:',F10.2,' PSI')
```

```
  460  FORMAT(1X,'THE NORMALIZED VERTICAL MOMENTS ARE:
      1(TOP-BOTTOM,LEFT-RIGHT)')
  470  FORMAT(1X,'VERT MOMENT NO.',5X,'NORMALIZED VALUE',5X,
      1'MAX PLASTIC MOMENT')
  480  FORMAT(8X,I3,9X,F17.4,5X,E15.4)
  485  FORMAT(1X,'HORZ MOMENT NO.',5X,'NORMALIZED VALUE',5X,
      1'MAX PLASTIC MOMENT')
  490  FORMAT(1X,'THE NORMALIZED HORIZONTAL MOMENTS ARE:
      1(LEFT-RIGHT,TOP-BOTTOM)')
C2345678901234567890123456789012345678901234567890123456789012345678901234567890
       STOP
       END


C234567

       SUBROUTINE DIMEN(A,AA,B,C,N,M1,M2,IA,IRW1,IW1,PSOL,
     1 DSOL,IW,RW,MN)
       DIMENSION AA(120,80),A(IA,N),B(IA),C(N),PSOL(MN)
       DIMENSION DSOL(IA),IW(IW1),RW(IRW1)
       DO 12 J=1,N
          DO 11 I=1,IA-2
             A(I,J)=AA(I,J)
   11     CONTINUE
   12  CONTINUE
       CALL ZX3LP(A,IA,B,C,N,M1,M2,S,PSOL,DSOL,RW,IW,IER)
       RETURN
       END
C
C      THIS SUBPROGRAM CALCULATES PLASTIC MOMENT OF EQUIVALENT
C      STIFFENER. WHEN CALLING THIS FUNCTION,"SPACE" IS THE
C      STIFFENER SPACING BETWEEN ADJACENT STIFFENERS.
```

```
C
      FUNCTION MP(DW,SPACE,T2,B1,T1,SIGMAY,T3)
      REAL MP
      R1=(DW*T3)/(SPACE*T2)
      R2=(B1*T1)/(SPACE*T2)
      R3=T2/DW
      R4=T1/DW
      IF(R1+R2.LT.0.99999) GO TO 5
      Z=SPACE*DW*T2*(R1*(R1+2.*R3+2.)+2.*R1*R2*(R4+1.)-(R2-1.)**2)
     1/(4.*R1)
      GO TO 6
    5 Z=SPACE*DW*T2*(2.*R1+4.*R2+R3*(1.+2.*R1+2.*R2)-R3*(R1+R2)**2
     1+2.*R2*R4)/4.
    6 MP=Z*SIGMAY
      RETURN
      END
C
C
C     THIS SUBPROGRAM CREATES BLANK LINES ON SCREEN
C
C

C234567
      FUNCTION SKIP(N)
      DO 10 I=1,N
         SKIP=I
         PRINT *, '  *'
```

```
   10 CONTINUE
      RETURN
      END
```

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ZX3LP LINEAR PROGRAM.

This section, taken from reference {15}, is intended to provide a brief explanation of the capabilities and uses of the linear programming subroutine ZX3LP.

```
IMSL ROUTINE NAME   - ZX3LP
PURPOSE             - SOLVE THE LINEAR PROGRAMMING PROBLEM
                        VIA THE REVISED SIMPLEX ALGORITHM,-
                        EASY TO USE VERSION
USAGE               - CALL ZX3LP (A,IA,B,C,N,M1,M2,S,PSOL,
                        DSOL,RW,IW,IER)
ARGUMENTS      A    - MATRIX OF DIMENSION M1+M2+2 BY N
                        CONTAINING THE COEFFICIENTS OF THE M1
                        INEQUALITY CONSTRAINTS IN THE FIRST M1
                        ROWS FOLLOWED BY THE COEFFICIENTS OF
                        THE M2 EQUALITY CONSTRAINTS. (INPUT)
                        THE LAST TWO ROWS OF A ARE USED ONLY
                        AS WORKING STORAGE.
               IA   - ROW DIMENSION OF MATRIX A EXACTLY AS
                        SPECIFIED IN THE DIMENSION STATEMENT
                        IN THE CALLING PROGRAM. (INPUT)
                        TWO ROWS OF A ARE REQUIRED FOR WORKING
                        STORAGE, AND THEREFORE, IA MUST
                        NOT BE LESS THAN M1+M2+2.
               B    - VECTOR OF LENGTH M1+M2+2 CONTAINING THE
                        RIGHT HAND SIDES OF THE INEQUALITY
                        CONSTRAINTS IN ITS FIRST M1 LOCATIONS
                        FOLLOWED BY THE M2 RIGHT HAND SIDES OF
                        THE EQUALITY CONSTRAINTS. (INPUT)
                        THE LAST TWO ELEMENTS OF B ARE USED
                        AS WORKING STORAGE.
               C    - VECTOR OF LENGTH N CONTAINING THE
                        COEFFICIENTS OF THE OBJECTIVE FUNCTION.
```

```
                    (INPUT)
          N    - NUMBER OF UNKNOWNS IN THE MODEL. (INPUT)
          M1   - NUMBER OF INEQUALITY CONSTRAINTS.(INPUT)
          M2   - NUMBER OF EQUALITY CONSTRAINTS.  (INPUT)
          S    - VALUE OF THE OBJECTIVE FUNCTION.(OUTPUT)
          PSOL - VECTOR OF LENGTH N CONTAINING THE PRIMAL
                   SOLUTION.  (OUTPUT) PSOL IS ALSO USED
                   AS WORK STORAGE AND THEREFORE MUST
                   HAVE LENGTH AT LEAST MAX(N,M1+M2).
          DSOL - VECTOR OF LENGTH M1+M2+2 CONTAINING THE
                   DUAL SOLUTION.  (OUTPUT)
                   THAT IS, DSOL(1)....,DSOL(M1+M2)
                   CONTAIN THE SOLUTION TO THE
                   PROBLEM MIN BT*Y SUBJECT TO AT*Y IS
                   GREATER THAN OR EQUAL TO C AND Y
                   GREATER THAN OR EQUAL TO 0 WHERE
                   AT=A-TRANSPOSE AND BT=B-TRANSPOSE.
                   WHEN THE PRIMAL PROBLEM HAS
                   EQUALITY CONSTRAINTS, THE
                   CORRESPONDING COMPONENTS OF THE
                   DUAL SOLUTION ARE UNCONSTRAINED.
                   DSOL(M1+M2+1) AND DSOL(M1+M2+2)
                   ARE USED AS WORKING STORAGE.
          RW   - WORK VECTOR OF LENGTH
                   (M1+M2+2)*(M1+M2+2)+ 3*M1+2*M2+4.
          IW   - WORK VECTOR OF LENGTH 2*M2+3*M1+4.
                IER  - ERROR INDICATOR. (OUTPUT)
                  TERMINAL ERROR
                   IER = 130 INDICATES THAT IA IS LESS
                     THAN M1+M2+2.
                   IER = 131 INDICATES THAT THE COST
                     CRITERION HAS UNBOUNDED VALUES.
                   IER = 132 INDICATES THAT THE MAXIMUM
                     NUMBER OF ITERATIONS WAS REACHED IN
                     ZX0LP.
                   IER = 133 INDICATES THAT NO FEASIBLE
                     SOLUTION EXISTS.
                  WARNING (WITH FIX)
                   IER = 70 INDICATES THAT SOME
                     ARTIFICIAL VARIABLES REMAINED IN THE
                     SOLUTION BASIS AT A ZERO LEVEL AFTER
                     PHASE 1. THIS CONDITION CAN BE CAUSED
                     BY HAVING REDUNDANT CONSTRAINTS.
                     NEVERTHELESS, A SOLUTION IS COMPUTED
                     AND RETURNED IN PSOL AND DSOL.
 PRECISION/HARDWARE - SINGLE AND DOUBLE/H32
                    - SINGLE/H36,H48,H60
 REQD. IMSL ROUTINES- UERTST,UGETIO,ZX0LP
```

NOTATION - INFORMATION ON SPECIAL NOTATION AND CONVENTIONS IS AVAILABLE IN THE MANUAL INTRODUCTION OR THROUGH IMSL ROUTINE UHELP.

REMARKS ZX3LP IS INTENDED TO VERY EASY TO USE. THEREFORE. THE CALLING PARAMETERS ARE VERY SIMPLE AND THERE ARE NO OPTIONS. ZX0LP CAN BE USED DIRECTLY IN SITUATIONS THAT REQUIRE MORE FLEXIBILITY THAN IS PROVIDED BY ZX3LP.

ALGORITHM

To solve the linear programming problem,

$$\text{maximize } c_1 PSOL_1 + \ldots + c_N PSOL_N = SP$$

subject to

$$a_{i1} PSOL_1 + \ldots + a_{iN} PSOL \leq b_i \qquad i=1,\ldots,M_1$$
$$a_{i1} PSOL_1 + \ldots + a_{iN} PSOL_N = b_i \qquad i=M_1+1,\ldots,M$$
$$PSOL_j \geq 0 \qquad j=1,\ldots,N$$

where $M = M_1 + M_2$.

The dual linear programming problem is,

$$\text{minimize } b_1 DSOL_1 + \ldots + b_M DSOL_M = SD$$

subject to

$$a_{1j} DSOL_1 + \ldots + a_{Mj} DSOL_M \geq c_j \qquad j=1,\ldots,N$$
$$DSOL_i \geq 0 \qquad i=1,\ldots,M_1$$
$$DSOL_i \text{ unrestricted in sign when } \quad i=M_1+1,\ldots,M$$

ZX3LP computes the solution to the primal problem, PSOL, the solution to the dual problem, DSOL, and the values of the objective function S=SP=SD.

ZX3LP calls ZX0LP which solves the linear programming problem by the revised simplex method.

See reference {14} for additional detail.

PROGRAMMING NOTES

ZX3LP is intended to very easy to use. Therefore, the calling parameters are very simple and there are no options.

ZX0LP can be used directly in situations that require more flexibility than is provided by ZX3LP.

EXAMPLE

Suppose we want to maximize $x_1 + 3x = S$

$$
\begin{aligned}
\text{subject to } x_1 \quad & \le 1 \\
x_2 & \le 1 \\
x_1 + x_2 & \le 1.5 \\
-x_1 - x_2 & \le -.5 \\
x_1 \ge 0, x_2 & \ge 0
\end{aligned}
$$

We can proceed as follows:

```
INTEGER   IA,N,M1,M2,IW(16),IER
REAL      A(6,2),B(6),C(2),RW(52),PSOL(4),DSOL(6),S
  N  =  2
  M1 =  4
  M2 =  0
  IA =  6
```

$$
A = \begin{bmatrix} 1. & 0. \\ 0. & 1. \\ 1. & 1. \\ -1. & -1. \end{bmatrix}
$$

$$
B = \begin{bmatrix} 1. \\ 1. \\ 1.5 \\ -.5 \end{bmatrix}
$$

$$
C = (1. \quad 3.)
$$

```
  CALL ZX3LP (A,IA,B,C,N,M1,M2,S,PSOL,DSOL,RW,IW,IER)
      .
      .
      .
  END
  Output
  IER = 0
  S   = 3.5
  PSOL = (.5   1.)
  DSOL = (0.  2.  1.  0.)
```

APPENDIX C SAMPLE OUTPUT

INPUT INFORMATION

NO. OF VERTICAL STIFFENERS: 10

NO. OF HORIZONTAL STIFFENERS: 10

PLATE DIAMETER: 10.00(FEET)

VERTICAL BEAMS
MATERIAL YIELD STRESS: 50000.00 PSI
PLATE THICKNESS: 2.000 INCHES
COMMON WEB THICKNESS: 0.000 INCHES
COMMON WEB DEPTH : 1.000 INCHES
COMMON BOTTOM FLANGE THICKNESS: 0.000 INCHES
COMMON BOTTOM FLANGE WIDTH : 0.000 INCHES

HORIZONTAL BEAMS
COMMON WEB THICKNESS: 0.000 INCHES
COMMON WEB DEPTH : 1.000 INCHES
COMMON BOTT.FLG.THICKNESS: 0.000 INCHES
COMMON BOTT.FLG. WIDTH : 0.000 INCHES

OUTPUT INFORMATION
UNSUPP. PLATE SECTION COLLAPSE LOAD:14722.2227
THE COLLAPSE PRESSURE IS: 154.70 PSI
THE NORMALIZED VERTICAL MOMENTS ARE:(TOP-BOTTOM,LEFT-RIGHT)

VERT MOMENT NO.	NORMALIZED VALUE	MAX PLASTIC MOMENT
1	-1.0000	0.5455E+06
2	0.0752	0.5455E+06
3	1.0000	0.5455E+06
4	1.0000	0.5455E+06
5	-1.0000	0.5455E+06
6	-0.0847	0.5455E+06
7	1.0000	0.5455E+06
8	1.0000	0.5455E+06
9	1.0000	0.5455E+06
10	-1.0000	0.5455E+06
11	-1.0000	0.5455E+06
12	1.0000	0.5455E+06
13	1.0000	0.5455E+06
14	0.8795	0.5455E+06
15	1.0000	0.5455E+06
16	-1.0000	0.5455E+06
17	-0.9338	0.5455E+06
18	-1.0000	0.5455E+06
19	-0.1566	0.5455E+06
20	1.0000	0.5455E+06

21	-0.6024	0.5455E+06
22	-1.0000	0.5455E+06
23	-0.3511	0.5455E+06
24	-0.0847	0.5455E+06
25	0.0571	0.5455E+06
26	-0.1988	0.5455E+06
27	1.0000	0.5455E+06

THE NORMALIZED HORIZONTAL MOMENTS ARE:(LEFT-RIGHT,TOP-BOTTOM)

HORZ MOMENT NO.	NORMALIZED VALUE	MAX PLASTIC MOMENT
1	-1.0000	0.5455E+06
2	0.7521	0.5455E+06
3	1.0000	0.5455E+06
4	1.0000	0.5455E+06
5	-1.0000	0.5455E+06
6	-0.8310	0.5455E+06
7	-0.8532	0.5455E+06
8	0.7270	0.5455E+06
9	1.0000	0.5455E+06
10	-1.0000	0.5455E+06
11	-1.0000	0.5455E+06
12	-0.6990	0.5455E+06
13	0.2891	0.5455E+06
14	1.0000	0.5455E+06
15	1.0000	0.5455E+06
16	-1.0000	0.5455E+06
17	-1.0000	0.5455E+06
18	-0.4728	0.5455E+06
19	-0.3433	0.5455E+06
20	-0.8523	0.5455E+06
21	1.0000	0.5455E+06
22	-1.0000	0.5455E+06
23	0.0480	0.5455E+06
24	0.7228	0.5455E+06
25	1.0000	0.5455E+06
26	1.0000	0.5455E+06
27	-1.0000	0.5455E+06

INPUT INFORMATION

NO. OF VERTICAL STIFFENERS: 4

NO. OF HORIZONTAL STIFFENERS: 5

PLATE DIAMETER: 12.00(FEET)

VERTICAL BEAMS
MATERIAL YIELD STRESS: 50000.00 PSI
PLATE THICKNESS: 1.000 INCHES
COMMON WEB THICKNESS: 0.800 INCHES
COMMON WEB DEPTH : 5.000 INCHES
COMMON BOTTOM FLANGE THICKNESS: 1.000 INCHES
COMMON BOTTOM FLANGE WIDTH : 4.000 INCHES

HORIZONTAL BEAMS
COMMON WEB THICKNESS: 0.750 INCHES
COMMON WEB DEPTH : 6.000 INCHES
COMMON BOTT.FLG.THICKNESS: 1.000 INCHES
COMMON BOTT.FLG. WIDTH : 4.000 INCHES

OUTPUT INFORMATION
UNSUPP. PLATE SECTION COLLAPSE LOAD: 588.3488
THE COLLAPSE PRESSURE IS: 142.43 PSI
THE NORMALIZED VERTICAL MOMENTS ARE:(TOP-BOTTOM,LEFT-RIGHT)

VERT MOMENT NO.	NORMALIZED VALUE	MAX PLASTIC MOMENT
1	-1.0000	0.2132E+07
2	-1.0000	0.2132E+07
3	-0.4645	0.2132E+07
4	1.0000	0.2132E+07
5	-1.0000	0.2132E+07
6	0.1584	0.2132E+07
7	1.0000	0.2132E+07
8	0.5362	0.2132E+07

THE NORMALIZED HORIZONTAL MOMENTS ARE:(LEFT-RIGHT,TOP-BOTTOM)

HORZ MOMENT NO.	NORMALIZED VALUE	MAX PLASTIC MOMENT
1	-1.0000	0.2450E+07
2	0.0455	0.2450E+07
3	1.0000	0.2450E+07
4	-1.0000	0.2450E+07
5	1.0000	0.2450E+07
6	1.0000	0.2450E+07
7	-1.0000	0.2450E+07
8	-0.8478	0.2450E+07
9	1.0000	0.2450E+07

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